Compute the first few terms of the Taylor series for $f(x) = \sec x$.

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$$f(x) = \sec x$$

$$f'(x) = \sec x \tan x$$

$$f''(x) = \sec x \tan^2 x + \sec^3 x$$

$$f'''(x) = \sec x \tan x \cdot \tan^2 x + \sec x \cdot 2\tan x \cdot \sec^2 x + 3\sec^2 x \cdot \sec x \tan x$$

$$= \sec \tan^3 x + 5\sec^3 x \cdot \sec x \tan x$$

$$f''''(x) = \sec x \tan x \cdot \tan^3 x + \sec x \cdot 3\tan^2 x \cdot \sec^2 x + 5 \cdot 4\sec^3 x \cdot \tan x$$
$$+ 5\sec^4 x \cdot \sec^2 x$$

$$= \sec x \tan^4 x + 3\sec^3 x \tan^2 x + 20\sec^3 x \tan x + 5\sec^6 x$$

$$\sec x \approx 1 + 0 + \frac{1}{2!}x^2 + 0 + \frac{5}{4!}x^4$$

$$= 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \cdots$$

Method 2:

$$\sec x \cos x = 1$$

$$\sec x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right) = 1$$

$$\sec x = a_0 + a_2x^2 + a_4x^4 + \cdots$$

$a_0 = 1$

$-\frac{a_0}{2} + a_2 = 0 \Rightarrow a_2 = \frac{1}{2}$

$\frac{a_0}{4!} - \frac{a_2}{2} + a_4 = 0 \Rightarrow \frac{1}{24} - \frac{1}{4} + a_4 = 0 \Rightarrow a_4 = \frac{5}{24}$

$\therefore \sec x = 1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 + \cdots$